Level 2

ELEMENTARY ALGEBRA and GEOMETRY READINESS DIAGNOSTIC TEST PRACTICE

Directions: Study the examples, work the problems, then check your answers at the end of each topic. If you don't get the answer given, check your work and look for mistakes. If you have trouble, ask a math teacher or someone else who understands this topic.

TOPIC 1: ARITHMETIC OPERATIONS

A. Fractions:

Simplifying fractions:

example: Reduce $\frac{27}{36}$: $\frac{27}{36}=\frac{9\bullet 3}{9\bullet 4}=\frac{9}{9}\bullet\frac{3}{4}=1\bullet\frac{3}{4}=\frac{3}{4}$

(Note that you must be able to find a common factor; in this case 9; in both the top and the bottom in order to reduce.)

Problems 1-3: Reduce:

1. $\frac{13}{52}=$ | 2. $\frac{26}{65}=$ | 3. $\frac{3+6}{3+9}=$

Equivalent fractions:

example: $\frac{3}{4}$ is equivalent to how many eighths?

$$\frac{3}{4}=\frac{}{8}$$
$$\frac{3}{4}=1\bullet\frac{3}{4}=\frac{2}{2}\bullet\frac{3}{4}=\frac{2\bullet 3}{2\bullet 4}=\frac{6}{8}$$

Problems 4-5: Complete:

4. $\frac{4}{9}=\frac{}{72}$ | 5. $\frac{3}{5}=\frac{}{20}$

How to get the lowest common denominator (LCD) by finding the least common multiple (LCM) of all denominators:

example: $\frac{5}{6}$ and $\frac{8}{15}$. First find LCM of 6 and 15:

$$6=2\bullet 3$$
$$15=3\bullet 5$$
$LCM=2\bullet 3\bullet 5=30$, so $\frac{5}{6}=\frac{25}{30}$, and $\frac{8}{15}=\frac{16}{30}$

Problems 6-7: Find equivalent fractions with the LCD:

6. $\frac{2}{3}$ and $\frac{2}{9}$ | 7. $\frac{3}{8}$ and $\frac{7}{12}$

8. Which is larger, $\frac{5}{7}$ or $\frac{3}{4}$?
(Hint: find the LCD fractions)

Adding, subtracting fractions: if the denominators are the same combine the numerators:

example: $\frac{7}{10}-\frac{1}{10}=\frac{7-1}{10}=\frac{6}{10}=\frac{3}{5}$

Problems 9-11: Find the sum or difference (reduce if possible):

9. $\frac{4}{7}+\frac{2}{7}=$ | 10. $\frac{5}{6}+\frac{1}{6}=$ | 11. $\frac{7}{8}-\frac{5}{8}=$

If the denominators are different, find equivalent fractions with common denominators, then proceed as before:

example: $\frac{4}{5}+\frac{2}{3}=\frac{12}{15}+\frac{10}{15}=\frac{22}{15}=1\frac{7}{15}$

example: $\frac{1}{2}-\frac{2}{3}=\frac{3}{6}-\frac{4}{6}=\frac{3-4}{6}=\frac{-1}{6}$

12. $\frac{3}{5}-\frac{2}{3}=$ | 13. $\frac{5}{8}+\frac{1}{4}=$

Multiplying fractions: multiply the top numbers, multiply the bottom numbers, reduce if possible.

example: $\frac{3}{4}\bullet\frac{2}{5}=\frac{3\bullet 2}{4\bullet 5}=\frac{6}{20}=\frac{3}{10}$

14. $\frac{2}{3}\bullet\frac{3}{8}=$

15. $\frac{1}{2}\bullet\frac{1}{3}=$

16. $\left(\frac{3}{4}\right)^2=$

17. $\left(2\frac{1}{2}\right)^2=$

Dividing fractions: make a compound fraction, then multiply the top and bottom (of the big fraction) by the LCD of both:

example: $\frac{3}{4}\div\frac{2}{3}=\frac{3/4}{2/3}=\frac{3/4\bullet 12}{2/3\bullet 12}=\frac{9}{8}$

example: $\frac{7}{\frac{2}{3}-\frac{1}{2}}=\frac{7\bullet 6}{\left(\frac{2}{3}-\frac{1}{2}\right)\bullet 6}=\frac{42}{4-3}=\frac{42}{1}=42$

18. $\frac{3}{2}\div\frac{1}{4}=$

19. $11\frac{3}{8}\div\frac{3}{4}=$

20. $\frac{3}{4}\div 2=$

21. $\frac{2}{3/4}=$

22. $\frac{2/3}{4}$

B. Decimals:

Meaning of places: in 324.519, each digit position has a value ten times the place to its right. The part to the left of the point is the whole number part. Right of the point, the places have values: tenths, hundredths, etc.,

So, $324.519=(3\times 100)+(2\times 10)+(4\times 1)+(5\times 1/10)+(1\times 1/100)+(9\times 1/1000)$.

23. Which is larger: .59 or .7?

To add or subtract decimals, like places must be combined (line up the points).

example: $1.23-.1=1.13$
example: $4+.3=4.3$
example: $6.04-(2-1.4)=6.04-.6=5.44$

24. $5.4+.78=$
25. $.36-.63=$

26. $4-.3+.001-.01+.1=$
27. $\$3.54-\$1.68=$

Multiplying decimals:

example:	$.3\times .5=.15$
example:	$.3\times .2=.06$
example:	$(.03)^2=.0009$

28. $3.24\times 10=$	30. $(.51)^2=$
29. $.01\times .2=$	31. $5\times .4=$

Dividing decimals: change the problem to an equivalent whole number problem by multiplying both by the same power of ten.

example: $.3\div .03$ Multiply both by 100, to get $30\div 3=10$
example: $\frac{.014}{.07}$ Multiply both by 1000, get $\frac{14}{70}=14\div 70=.2$

32. $.013\div 100=$	34. $\frac{340}{3.4}=$
33. $.053\div .2=$	

C. Positive integer exponents and square roots of perfect squares:

Meaning of exponents (powers):

example:	$3^4=3\bullet 3\bullet 3\bullet 3=81$
example:	$4^3=4\bullet 4\bullet 4=64$

Problems 35-44: Find the value:

35. $3^2=$	40. $100^2=$
36. $(-3)^{2=}$	41. $(2.1)^2=$
37. $-(3)^2=$	42. $(-.1)^3=$
38. $-3^2=$	43. $\left(\frac{2}{3}\right)^3=$
39. $(-2)^3=$	44. $\left(-\frac{2}{3}\right)^3=$

$\sqrt{a}$ is a non-negative real number if $a\ge 0$

$\sqrt{a}=b$ means $b^2=a$, where $b\ge 0$. Thus $\sqrt{49}=7$, because $7^2=49$. Also, $-\sqrt{49}=-7$.

45. $\sqrt{144}=$	49. $\sqrt{1.44}=$
46. $-\sqrt{144}=$	50. $\sqrt{.09}=$
47. $\sqrt{-144}=$	51. $\sqrt{\frac{4}{9}}=$
48. $\sqrt{8100}=$	

D. Fraction-decimal conversion:

Fraction to decimal: divide the top by the bottom.

example: $\frac{3}{4}=3\div 4=.75$
example: $\frac{20}{3}=20\div 3=6.66666...=6.\overline{6}$
example: $3\frac{2}{5}=3+\frac{2}{5}=3+(2\div 5)=3+.4=3.4$

Problems 52-55: Write each as a decimal. If the decimal repeats, show the repeating block of digits:

52. $\frac{5}{8}=$	54. $4\frac{1}{3}=$
53. $\frac{3}{7}=$	55. $\frac{3}{100}=$

Non-repeating decimals to fractions: read the number as a fraction, write it as a fraction, reduce if possible:

example: $.4=$ four tenths $=\frac{4}{10}=\frac{2}{5}$
example: $3.76=$ three and seventy six hundredths $=3\frac{76}{100}=3\frac{19}{25}$

Problems 56-58: Write as a fraction:

56. $.01=$	57. $4.9=$	58. $1.25=$

E. Percents:

Meaning of percent: translate 'percent' as 'hundredths':

example: 8% means 8 hundredths or .08 or $\frac{8}{100}=\frac{2}{25}$

To change a decimal to percent form, multiply by 100: move the point 2 places right and write the % symbol.

example: $.075=7.5\%$
example: $1\frac{1}{4}=1.25=125\%$

Problems 59-60: Write as a percent:

59. $.3=$	60. $4=$

To change a percent to decimal form, move the point 2 places left and drop the % symbol.

example: $8.76\%=.0876$
example: $67\%=.67$

Problems 61-62: Write as a decimal:

61. $10\%=$	62. $.03\%=$

To solve a percent problem which can be written in this form: a % of b is c
First identify a,b,c:

Problems 63-65: If each statement were written (with the same meaning) in the form of a % of b is c, identify a, b, and c:

63. 3% of 40 is 1.2
64. 600 is 150% of 400
65. 3 out of 12 is 25%

Given a and b, change $a\%$ to decimal form and multiply (since 'of' can be translated 'multiply').

Given c and one of the others, divide c by the other (first change percent to decimal, or if the answer is a, write it as a percent).

example: What is 9.4% of \$5000? ($a\%$ of b is c: 9.4% of \$5000 is __?__)

$9.4\% = .094$

$.094 \times \$5000 = \$470$ (answer)

example: 56 problems correct out of 80 is what percent? (a % of b is c: __?__% of 80 is 56)

$56 \div 80 = .7 = 70\%$ (answer)

example: 5610 people vote in an election, which is 60% of the registered voters. How many are registered? (a % of b is c: 60 % of __?__ is 5610);

$60\% = .6;\ \ 5610 \div .6 = 9350$ (answer)

66. 4% of 9 is what?
67. What percent of 70 is 56?
68. 15% of what is 60?

F. <u>Estimation and approximation</u>:

<u>Rounding to one significant digit</u>:

example: 3.67 rounds to 4
example: .0449 rounds to .04
example: 850 rounds to either 800 or 900

Problems 69-71: Round to one significant digit.

69. 45.01 | 70. 1.09 | 71. 00083

<u>To estimate an answer</u>, it is often sufficient to round each given number to one significant digit, then compute.

example: $.0298 \times .000513$
Round and compute: $.03 \times .0005 = .000015$
.000015 is the estimate

Problems 72-75: Select the best approximation of the answer:

72. $1.2346825 \times 367.0003246 =$ (4, 40, 400, 4000, 40000)
73. $.0042210398 \div .0190498238 =$ (.02, .2, .5, 5, 20, 50)
74. $101.7283507 + 3.141592653 =$ (2, 4, 98, 105, 400)
75. $(4.36285903)^3 =$ (12, 64, 640, 5000, 12000)

<u>Answers</u>:

1. $\frac{1}{4}$
2. $\frac{2}{5}$
3. $\frac{3}{4}$
4. 32
5. 12
6. $\frac{6}{9}$, $\frac{2}{9}$
7. $\frac{9}{24}$, $\frac{14}{24}$
8. $\frac{3}{4}$ (because $\frac{20}{28} < \frac{21}{28}$)
9. $\frac{6}{7}$
10. 1
11. $\frac{1}{4}$
12. $-\frac{1}{15}$
13. $\frac{7}{8}$
14. $\frac{1}{4}$
15. $\frac{1}{6}$
16. $\frac{9}{16}$
17. $\frac{25}{4}$
18. 6
19. 15 $\frac{1}{6}$
20. $\frac{3}{8}$
21. $\frac{8}{3}$
22. $\frac{1}{6}$
23. .7
24. 6.18
25. -.27
26. 3.791
27. \$1.86
28. 32.4
29. .002
30. .2601
31. 2
32. .00013
33. .265
34. 100
35. 9
36. 9
37. –9
38. –9
39. –8
40. 10000
41. 4.41
42. -.001
43. $\frac{8}{27}$
44. $-\frac{8}{27}$
45. 12
46. –12
47. not a real number
48. 90
49. 1.2
50. .3
51. $\frac{2}{3}$
52. .625
53. $.\overline{428571}$
54. $4.\overline{3}$
55. .03
56. $\frac{1}{100}$
57. $4\frac{9}{10} = \frac{49}{10}$
58. $1\frac{1}{4} = \frac{5}{4}$
59. 30%
60. 400%
61. .1
62. .0003

	a	b	c
63.	3	40	1.2
64.	150	400	600
65.	25	12	3

66. .36
67. 80%
68. 400
69. 50
70. 1
71. .0008
72. 400
73. .2
74. 105
75. 64

TOPIC 2: POLYNOMIALS

A. <u>Grouping to simplify polynomials</u>:

The distributive property says: $a(b+c) = ab + ac$

example: $3(x-y) = 3x - 3y$
$(a = 3,\ b = x,\ c = -y)$

example:	$4x+7x=(4+7)x=11x$ $(a=x,\ b=4,\ c=7)$
example:	$4a+6x-2=2(2a+3x-1)$

Problems 1-3: Rewrite, using the distributive property:

1. $6(x-3)=$
2. $4x-x=$
3. $-5(a-1)=$

Commutative and associative properties are also used in regrouping:

example: $3x+7-x=3x-x+7=2x+7$
example: $5-x+5=5+5-x=10-x$
example: $3x+2y-2x+3y$
$=3x-2x+2y+3y=x+5y$

Problems 4-9: Simplify:

4. $x+x=$
5. $a+b-a+b=$
6. $9x-y+3y-8x=$
7. $4x+1+x-2=$
8. $180-x-90=$
9. $x-2y+y-2x=$

B. <u>Evaluation by substitution</u>:

example: If $x=3$, then
$7-4x=7-4(3)=7-12=-5$
example: If $a=-7$ and $b=-1$, then
$a^2b=(-7)^2(-1)=49(-1)=-49$
example: If $x=-2$, then $3x^2-x-5=$
$3(-2)^2-(-2)-5=3\bullet 4+2-5=$
$12+2-5=9$

Problems 10-19: Given $x=-1$, $y=3$, and $z=-3$. Find the value:

10. $2x=$
11. $-z=$
12. $xz=$
13. $y+z=$
14. $y^2+z^2=$
15. $2x+4y=$
16. $2x^2-x-1=$
17. $(x+z)^2=$
18. $x^2+z^2=$
19. $-x^2z=$

C. <u>Adding, subtracting polynomials</u>:
Combine like terms:

example: $(3x^2+x+1)-(x-1)=$
$3x^2+x+1-x+1=3x^2+2$
example: $(x-1)+(x^2+2x-3)=$
$x-1+x^2+2x-3=x^2+3x-4$
example: $(x^2+x-1)-(6x^2-2x+1)=$
$x^2+x-1-6x^2+2x-1=-5x^2+3x-2$

Problems 20-25: Simplify:

20. $(x^2+x)-(x+1)=$
21. $(x-3)+(5-2x)=$
22. $(2a^2-a)+(a^2+a-1)=$
23. $(y^2-3y-5)-(2y^2-y+5)=$
24. $(7-x)-(x-7)=$
25. $x^2-(x^2+x-1)=$

D. <u>Monomial times polynomial</u>:
Use the distributive property:

example: $3(x-4)=3\bullet x+3(-4)=$
$3x+(-12)=3x-12$
example: $(2x+3)a=2ax+3a$
example: $-4x(x^2-1)=-4x^3+4x$

26. $-(x-7)=$
27. $-2(3-a)=$
28. $x(x+5)=$
29. $(3x-1)7=$
30. $a(2x-3)=$
31. $(x^2-1)(-1)=$
32. $8(3a^2+2a-7)=$

E. <u>Multiplying polynomials</u>:
Use the distributive property: $a(b+c)=ab+ac$

example: $(2x+1)(x-4)$ is $a(b+c)$ if:
$a=(2x+1)$, $b=x$, and $c=-4$
So $a(b+c)=ab+ac$
$=(2x+1)x+(2x+1)(-4)$
$=2x^2+x-8x-4=2x^2-7x-4$

Short cut to multiply above two binomials (see example above): FOIL (do mentally and write the answer)
F: <u>F</u>irst times <u>F</u>irst: $(2x)(x)=2x^2$
O: multiply '<u>O</u>uters': $(2x)(-4)=-8x$
I: multiply '<u>I</u>nners': $(1)(x)=x$
L: <u>L</u>ast times <u>L</u>ast: $(1)(-4)=-4$
Add, get $2x^2-7x-4$

example: $(x+2)(x+3)=x^2+5x+6$
example: $(2x-1)(x+2)=2x^2+3x-2$
example: $(x-5)(x+5)=x^2-25$
example: $-4(x-3)=-4x+12$
example: $(3x-4)^2=9x^2-24x+16$
example: $(x+3)(a-5)=ax-5x+3a-15$

Problems 33-41: Multiply:

33. $(x+3)^2=$
34. $(x-3)^2=$
35. $(x+3)(x-3)=$
36. $(2x+3)(2x-3)=$
37. $(x-4)(x-2)=$
38. $-6x(3-x)=$
39. $\left(x-\frac{1}{2}\right)^2=$
40. $(x-1)(x+3)=$
41. $(x^2-1)(x^2+3)=$

F. Special products:

These product patterns (examples of FOIL) should be remembered and recognized:

I. $(a+b)(a-b)=a^2-b^2$

II. $(a+b)^2=a^2+2ab+b^2$

III. $(a-b)^2=a^2-2ab+b^2$

Problems 42-44: Match each pattern with its example:

a. $(3x-1)^2=9x^2-6x+1$

b. $(x+5)^2=x^2+10x+25$

c. $(x+8)(x-8)=x^2-64$

42. I:	43. II:	44. III:

Problems 45-52: Write the answer using the appropriate product pattern:

45. $(3a+1)(3a-1)=$	49. $(3a-2)(3a-2)=$
46. $(y-1)^2=$	50. $(x-y)^2=$
47. $(3a+2)^2=$	51. $(4x+3y)^2=$
48. $(3a+2)(3a-2)=$	52. $(3x+y)(3x-y)=$

G. Factoring:

Monomial factors: $ab+ac=a(b+c)$

example: $x^2-x=x(x-1)$

example: $4x^2y+6xy=2xy(2x+3)$

Difference of two squares:

$a^2-b^2=(a+b)(a-b)$

example: $9x^2-4=(3x+2)(3x-2)$

Trinomial square:

$a^2+2ab+b^2=(a+b)^2$

$a^2-2ab+b^2=(a-b)^2$

example: $x^2-6x+9=(x-3)^2$

Trinomial:

example: $x^2-x-2=(x-2)(x+1)$

example: $6x^2-7x-3=(3x+1)(2x-3)$

Problems 53-67: Factor:

53. $a^2+ab=$	61. $x^2-3x-10=$
54. $a^3-a^2b+ab^2=$	62. $2x^2-x=$
55. $8x^2-2=$	63. $8x^3+8x^2+2x=$
56. $x^2-10x+25=$	64. $9x^2+12x+4=$
57. $-4xy+10x^2=$	65. $6x^3y^2-9x^4y=$
58. $2x^2-3x-5=$	66. $1-x-2x^2=$
59. $x^2-x-6=$	67. $3x^2-10x+3=$
60. $x^2y-y^2x=$	

Answers:

1. $6x-18$
2. $3x$
3. $-5a+5$
4. $2x$
5. $2b$
6. $x+2y$
7. $5x-1$
8. $90-x$
9. $-x-y$
10. -2
11. 3
12. 3
13. 0
14. 18
15. 10
16. 2
17. 16
18. 10
19. 3
20. x^2-1
21. $2-x$
22. $3a^2-1$
23. $-y^2-2y-10$
24. $14-2x$
25. $-x+1$
26. $-x+7$
27. $-6+2a$
28. x^2+5x
29. $21x-7$
30. $2ax-3a$
31. $-x^2+1$
32. $24a^2+16a-56$
33. x^2+6x+9
34. x^2-6x+9
35. x^2-9
36. $4x^2-9$
37. x^2-6x+8
38. $-18x+6x^2$
39. $x^2-x+\frac{1}{4}$
40. x^2+2x-3
41. x^4+2x^2-3
42. c
43. b
44. a
45. $9a^2-1$
46. y^2-2y+1
47. $9a^2+12a+4$
48. $9a^2-4$
49. $9a^2-12a+4$
50. $x^2-2xy+y^2$
51. $16x^2+24xy+9y^2$
52. $9x^2-y^2$
53. $a(a+b)$
54. $a(a^2-ab+b^2)$
55. $2(2x+1)(2x-1)$
56. $(x-5)^2$
57. $-2x(2y-5x)$
58. $(2x-5)(x+1)$
59. $(x-3)(x+2)$
60. $xy(x-y)$
61. $(x-5)(x+2)$
62. $x(2x-1)$
63. $2x(2x+1)^2$
64. $(3x+2)^2$
65. $3x^3y(2y-3x)$
66. $(1-2x)(1+x)$
67. $(3x-1)(x-3)$

TOPIC 3: LINEAR EQUATIONS and INEQUALITIES

A. <u>Solving one linear equation in one variable</u>: Add or subtract the same value on each side of the equation, or multiply or divide each side by the same value, with the goal of placing the variable alone on one side. If there are one or more fractions, it may be desirable to eliminate them by multiplying both sides by the common denominator. If the equation is a proportion, you may wish to cross-multiply.

Problems 1-11: Solve:

1. $2x = 9$
2. $3 = \frac{6x}{5}$
3. $3x + 7 = 6$
4. $\frac{x}{3} = \frac{5}{4}$
5. $5 - x = 9$
6. $x = \frac{2x}{5} + 1$
7. $4x - 6 = x$
8. $x - 4 = \frac{x}{2} + 1$
9. $6 - 4x = x$
10. $7x - 5 = 2x + 10$
11. $4x + 5 = 3 - 2x$

To solve a linear equation for one variable in terms of the other(s), do the same as above:

example: Solve for F: $C = \frac{5}{9}(F - 32)$
Multiply by $\frac{9}{5}$: $\frac{9}{5}C = F - 32$
Add 32: $\frac{9}{5}C + 32 = F$
Thus, $F = \frac{9}{5}C + 32$
example: Solve for b: $a + b = 90$
Subtract a: $b = 90 - a$
example: Solve for x: $ax + b = c$
Subtract b: $ax = c - b$
Divide by a: $x = \frac{c-b}{a}$

Problems 12-19: Solve for the indicated variable in terms of the other(s):

12. $a + b = 180$
    $b =$
13. $2a + 2b = 180$
    $b =$
14. $P = 2b + 2h$
    $b =$
15. $y = 3x - 2$
    $x =$
16. $y = 4 - x$
    $x =$
17. $y = \frac{2}{3}x + 1$
    $x =$
18. $ax + by = 0$
    $x =$
19. $by - x = 0$
    $y =$

B. <u>Solution of a one-variable equation reducible to a linear equation</u>: Some equations which do not appear to be linear can be solved by using a related linear equation:

example: $\frac{x+1}{3x} = -1$
Multiply by $3x$: $x + 1 = -3x$
Solve: $4x = -1$
$x = -\frac{1}{4}$
(Be sure to check answer in the original equation.)
example: $\frac{3x+3}{x+1} = 5$
Think of 5 as $\frac{5}{1}$ and cross-multiply:
$5x + 5 = 3x + 3$
$2x = -2$
$x = -1$
But $x = -1$ does not make the original equation true (thus it does not check), so there is <u>no</u> solution.

Problems 20-25: Solve and check:

20. $\frac{x-1}{x+1} = \frac{6}{7}$
21. $\frac{3x}{2x+1} = \frac{5}{2}$
22. $\frac{3x-2}{2x+1} = 4$
23. $\frac{x+3}{2x} = 2$
24. $\frac{1}{3} = \frac{x}{x+8}$
25. $\frac{x-2}{4-2x} = 3$

example: $|3 - x| = 2$
Since the absolute value of both 2 and -2 is 2, $3 - x$ can be either 2 or -2. Write these two equations and solve each:
$3 - x = 2$ or $3 - x = -2$
$-x = -1$ $\quad -x = -5$
$x = 1$ <u>or</u> $x = 5$

Problems 26-30: Solve:

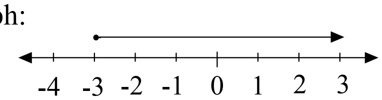
26. $|x| = 3$
27. $|x| = -1$
28. $|x - 1| = 3$
29. $|2 - 3x| = 0$
30. $|x + 2| = 1$

C. <u>Solution of linear inequalities</u>:

Rules for inequalities:

If $a > b$, then:	If $a < b$, then:
$a + c > b + c$	$a + c < b + c$
$a - c > b - c$	$a - c < b - c$
$ac > bc$ (if $c > 0$)	$ac < bc$ (if $c > 0$)
$ac < bc$ (if $c < 0$)	$ac > bc$ (if $c < 0$)
$\frac{a}{c} > \frac{b}{c}$ (if $c > 0$)	$\frac{a}{c} < \frac{b}{c}$ (if $c > 0$)
$\frac{a}{c} < \frac{b}{c}$ (if $c < 0$)	$\frac{a}{c} > \frac{b}{c}$ (if $c < 0$)

example: One variable graph: solve and graph on a number line: $1 - 2x \le 7$ (This is an abbreviation for $\{x : 1 - 2x \le 7\}$)
Subtract 1, get $-2x \le 6$
Divide by -2, $x \ge -3$
Graph:



Problems 31-38: Solve and graph on a number line:

31. $x - 3 > 4$
32. $4x < 2$

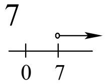
33. $2x+1 \le 6$
34. $3 < x-3$
35. $4-2x < 6$
36. $5-x > x-3$
37. $x > 1+4$
38. $6x+5 \ge 4x-3$

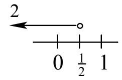
D. <u>Solving a pair of linear equations in two variables</u>:
The solution consists of an ordered pair, an infinite number of ordered pairs, or no solution.

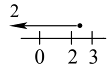
Problems 39-46: Solve for the common solution(s) by substitution or linear combinations:

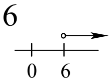
39. $x+2y=7$
$3x-y=28$
40. $x+y=5$
$x-y=-3$
41. $2x-y=-9$
$x=8$
42. $2x-y=1$
$y=x-5$
43. $2x-3y=5$
$3x+5y=1$
44. $4x-1=y$
$4x+y=1$
45. $x+y=3$
$x+y=1$
46. $2x-y=3$
$6x-9=3y$

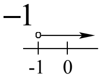
<u>Answers</u>:

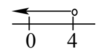
1. $9/2$
2. $5/2$
3. $-1/3$
4. $15/4$
5. -4
6. $5/3$
7. 2
8. 10
9. $6/5$
10. 3
11. $-1/3$
12. $180-a$
13. $90-a$
14. $\frac{(P-2h)}{2}$
15. $\frac{(y+2)}{3}$
16. $4-y$
17. $\frac{(3y-3)}{2}=\frac{3(y-1)}{2}$
18. $-by/a$
19. x/b
20. 13
21. $-5/4$
22. $-6/5$
23. 1
24. 4
25. no solution
26. –3, 3
27. no solution
28. –2, 4
29. $2/3$
30. –3, –1
31. $x>7$


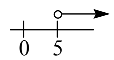
32. $x<\frac{1}{2}$


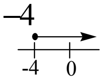
33. $x \le \frac{5}{2}$


34. $x>6$


35. $x>-1$


36. $x<4$


37. $x>5$


38. $x \ge -4$


39. $(9,-1)$
40. (1, 4)
41. (8, 25)
42. $(-4,-9)$
43. $(28/19, -13/19)$
44. $(1/4, 0)$
45. no solution
46. any ordered pair of the form $(a, 2a-3)$ where a is any number. One example: $(4,5)$. Infinitely many solutions.

TOPIC 4: QUADRATIC EQUATIONS

A. $\underline{ax^2+bx+c=0}$:
A quadratic equation can always be written so it looks like $ax^2+bx+c=0$ where a, b, and c are real numbers and a is not zero.

example: $5-x=3x^2$
Add x: $5=3x^2+x$
Subtract 5: $0=3x^2+x-5$
or $3x^2+x-5=0$
So $a=3$, $b=1$, $c=-5$

example: $x^2=3$
Rewrite: $x^2-3=0$
(Think of $x^2+0x-3=0$)
So $a=1$, $b=0$, $c=-3$

Problems 1-4: Write each of the following in the form $ax^2+bx+c=0$ and identify a, b, c:

1. $3x+x^2-4=0$
2. $5-x^2=0$
3. $x^2=3x-1$
4. $x=3x^2$
5. $81x^2=1$

B. <u>Factoring</u>:

<u>Monomial factors</u>:
$ab+ac=a(b+c)$

example: $x^2-x=x(x-1)$
example: $4x^2y+6xy=2xy(2x+3)$

<u>Difference of two squares</u>:
$a^2-b^2=(a+b)(a-b)$

example: $9x^2-4=(3x+2)(3x-2)$

Trinomial square:

$a^2 + 2ab + b^2 = (a+b)^2$

$a^2 - 2ab + b^2 = (a-b)^2$

example: $x^2 - 6x + 9 = (x-3)^2$

Trinomial:

example: $x^2 - x - 2 = (x-2)(x+1)$
example: $6x^2 - 7x - 3 = (3x+1)(2x-3)$

Problems 6-20: Factor:

6. $a^2 + ab =$	14. $x^2 - 3x - 10 =$
7. $a^3 - a^2b + ab^2 =$	15. $2x^2 - x$
8. $8x^2 - 2 =$	16. $2x^3 + 8x^2 + 8x =$
9. $x^2 - 10x + 25 =$	17. $9x^2 + 12x + 4 =$
10. $-4xy + 10x^2 =$	18. $6x^3y^2 - 9x^4y =$
11. $2x^2 - 3x - 5 =$	19. $1 - x - 2x^2 =$
12. $x^2 - x - 6 =$	20. $3x^2 - 10x + 3 =$
13. $x^2y - y^2x =$	

C. Solving factored quadratic equations:
The following statement is the central principle:

If $ab = 0$, then $a = 0$ or $b = 0$.

First, identify *a* and *b* in $ab = 0$:

example: $(3-x)(x+2) = 0$
Compare this with $ab = 0$
$a = (3-x)$; $b = (x+2)$

Problems 21-24: Identify *a* and *b* in each of the following:

21. $3x(2x-5) = 0$	23. $(2x-1)(x-5) = 0$
22. $(x-3)x = 0$	24. $0 = (x-1)(x+1)$

Then, because $ab = 0$ means $a = 0$ or $b = 0$, we can use the factors to make two linear equations to solve:

> *example:* If $2x(3x-4) = 0$ then $(2x) = 0$ or $(3x-4) = 0$
> and so $x = 0$ or $3x = 4$; $x = \frac{4}{3}$.
> Thus, there are two solutions: 0 and $\frac{4}{3}$
>
> *example:* If $(3-x)(x+2) = 0$ then $(3-x) = 0$ or $(x+2) = 0$ and thus $x = 3$ or $x = -2$.
>
> *example:* If $(2x+7)^2 = 0$
> then $2x + 7 = 0$
> $2x = -7$
> $x = -\frac{7}{2}$ (one solution)

Note: there must be a zero on one side of the equation to solve by the factoring method.

Problems 25-31: Solve:

25. $(x+1)(x-1) = 0$	29. $(x-6)(x-6) = 0$
26. $4x(x+4) = 0$	30. $(2x-3)^2 = 0$
27. $0 = (2x-5)x$	31. $x(x+2)(x-3) = 0$
28. $0 = (2x+3)(x-1)$	

D. Solving quadratic equations by factoring:
Arrange the equation so zero is on one side (in the form $ax^2 + bx + c = 0$), factor, set each factor equal to zero, and solve the resulting linear equations.

> *example:* Solve: $6x^2 = 3x$
> Rewrite: $6x^2 - 3x = 0$
> Factor: $3x(2x-1) = 0$
> So $3x = 0$ or $(2x-1) = 0$
> Thus, $x = 0$ or $x = \frac{1}{2}$
>
> *example:* $0 = x^2 - x - 12$
> $0 = (x-4)(x+3)$
> $x - 4 = 0$ or $x + 3 = 0$
> $x = 4$ or $x = -3$

Problems 32-43: Solve by factoring:

32. $x(x-3) = 0$	38. $0 = (x+2)(x-3)$
33. $x^2 - 2x = 0$	39. $(2x+1)(3x-2) = 0$
34. $2x^2 = x$	40. $6x^2 = x + 2$
35. $3x(x+4) = 0$	41. $9 + x^2 = 6x$
36. $x^2 = 2 - x$	42. $1 - x = 2x^2$
37. $x^2 + x = 6$	43. $x^2 - x - 6 = 0$

Another problem form: if a problem is stated in this form: 'One of the solutions of $ax^2 + bx + c = 0$ is *d*', solve the equation as above, then verify the statement.

> *example:* Problem: One of the solutions of $10x^2 - 5x = 0$ is
> A. -2
> B. $-\frac{1}{2}$
> C. $\frac{1}{2}$
> D. 2
> E. 5
>
> Solve $10x^2 - 5x = 0$ by factoring:
> $5x(2x-1) = 0$ so $5x = 0$ or $(2x-1) = 0$
> thus $x = 0$ or $x = \frac{1}{2}$.
>
> Since $x = \frac{1}{2}$ is one solution, answer C is correct.

44. One of the solutions of $(x-1)(3x+2)=0$ is
  A. $-\frac{3}{2}$
  B. $-\frac{2}{3}$
  C. 0
  D. $\frac{2}{3}$
  E. $\frac{3}{2}$

45. One solution of $x^2-x-2=0$ is
  A. -2
  B. -1
  C. $-\frac{1}{2}$
  D. $\frac{1}{2}$
  E. 1

Answers:

	a	b	c
1. $x^2+3x-4=0$	1	3	-4
2. $-x^2+5=0$	-1	0	5
3. $x^2-3x+1=0$	1	-3	1
4. $3x^2-x=0$	3	-1	0
5. $81x^2-1=0$	81	0	-1

Note: all signs could be the opposite.

6. $a(a+b)$
7. $a(a^2-ab+b^2)$
8. $2(2x+1)(2x-1)$
9. $(x-5)^2$
10. $-2x(2y-5x)$
11. $(2x-5)(x+1)$
12. $(x-3)(x+2)$
13. $xy(x-y)$
14. $(x-5)(x+2)$
15. $x(2x-1)$
16. $2x(x+2)^2$
17. $(3x+2)^2$
18. $3x^3y(2y-3x)$
19. $(1-2x)(1+x)$
20. $(3x-1)(x-3)$

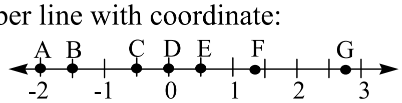
	a	b
21.	$3x$	$2x-5$
22.	$x-3$	x
23.	$2x-1$	$x-5$
24.	$x-1$	$x+1$

25. -1, 1
26. -4, 0
27. 0, $\frac{5}{2}$
28. $-\frac{3}{2}$, 1
29. 6
30. $\frac{3}{2}$
31. -2, 0, 3
32. 0, 3
33. 0, 2
34. 0, $\frac{1}{2}$
35. -4, 0
36. -2, 1
37. -3, 2
38. -2, 3
39. $-\frac{1}{2}$, $\frac{2}{3}$
40. $-\frac{1}{2}$, $\frac{2}{3}$
41. 3
42. -1, $\frac{1}{2}$
43. -2, 3
44. B
45. B

TOPIC 5: GRAPHING

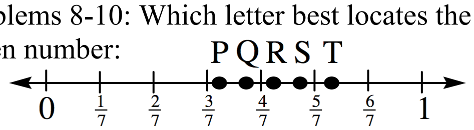
A. <u>Graphing a point on the number line</u>:

Problems 1-7: Select the letter of the point on the number line with coordinate:



1. 0
2. $\frac{1}{2}$
3. $-\frac{1}{2}$
4. $\frac{4}{3}$
5. -1.5
6. 2.75
7. $-\frac{3}{2}$

Problems 8-10: Which letter best locates the given number:

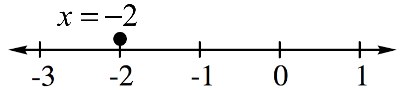


8. $\frac{5}{9}$
9. $\frac{3}{4}$
10. $\frac{2}{3}$

Problems 11-13: Solve each equation and graph the solution on the number line:

example: $x+3=1$

$x=-2$



11. $2x-6=0$
12. $x=3x+5$
13. $4-x=3+x$

B. <u>Graphing a linear inequality (in one variable) on the number line</u>:

Rules for inequalities:

If $a>b$, then:	If $a<b$, then:
$a+c>b+c$	$a+c<b+c$
$a-c>b-c$	$a-c<b-c$
$ac>bc$ (if $c>0$)	$ac<bc$ (if $c>0$)
$ac<bc$ (if $c<0$)	$ac>bc$ (if $c<0$)
$\frac{a}{c}>\frac{b}{c}$ (if $c>0$)	$\frac{a}{c}<\frac{b}{c}$ (if $c>0$)
$\frac{a}{c}<\frac{b}{c}$ (if $c<0$)	$\frac{a}{c}>\frac{b}{c}$ (if $c<0$)

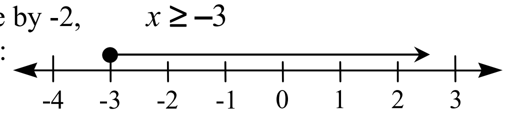
example: One variable graph: solve and graph on a number line: $1-2x\le 7$

(This is an abbreviation for $\{x: 1-2x\le 7\}$)

Subtract 1, get $-2x\le 6$

Divide by -2, $x\ge -3$

Graph:



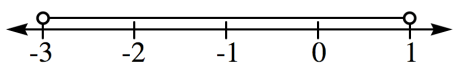
Problems 14-20: Solve and graph on a number line:

14. $x-3>4$
15. $4x<2$
16. $2x+1\le 6$
17. $3<x-3$
18. $4-2x<6$
19. $5-x>x-3$
20. $x>1+4$

example: $x>-3$ <u>and</u> $x<1$
The two numbers -3 and 1 splits the number line into three parts: $x>-3$, $-3<x<1$, and $x<1$. Check each part to see if <u>both</u> $x>-3$ <u>and</u> $x<1$ are true:

part	x values	$x>-3$?	$x<1$?	both true?
1	$x<-3$	no	yes	no
2	$-3<x<1$	yes	yes	yes (solution)
3	$x>1$	yes	no	no

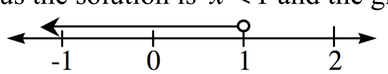
Thus the solution is $-3<x<1$ and the graph is:



example: $x\le -3$ or $x<1$
('or' means 'and/or')

part	x values	$x\le -3$?	$x<1$?	at least one true?
1	$x\le -3$	yes	yes	yes (solution)
2	$-3\le x<1$	no	yes	yes (solution)
3	$x>1$	no	no	no

So $x\le -3$ or $-3\le x<1$; these cases are both covered if $x<1$. Thus the solution is $x<1$ and the graph is:



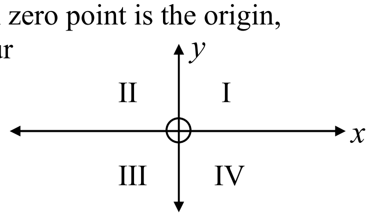
Problems 21-23: Solve and graph:

21. $x<1$ or $x>3$
22. $x\ge 0$ and $x>2$
23. $x>1$ and $x\le 4$

C. <u>Graphing a point in the coordinate plane</u>:

If two number lines intersect at right angles so that:
1) one is horizontal with positive to the right and negative to the left,
2) the other is vertical with positive up and negative down, and
3) the zero points coincide

Then they form a coordinate plane, and
1) the horizontal number line is called the *x*-axis,
2) the vertical line is the *y*-axis,
3) the common zero point is the origin,
4) there are four quadrants, numbered as shown:



To locate a point on the plane, an ordered pair of numbers is used, written in the form (*x*, *y*).
The *x*-coordinate is always given first.

Problems 24-27: Identify *x* and *y* in each ordered pair:

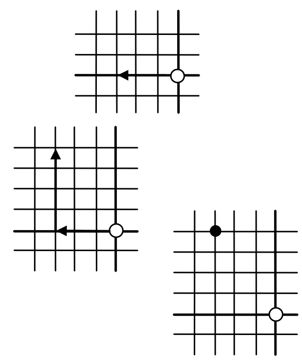
24. (3, 0)
25. (-2, 5)
26. (5, -2)
27. (0, 3)

To plot a point, start at the origin and make the moves, first in the *x*-direction (horizontal) and the in the *y*-direction (vertical) indicated by the ordered pair.

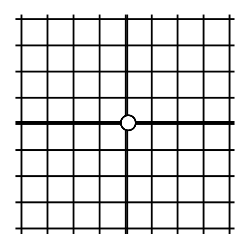
example: (-3, 4)
Start at the origin, move left 3 (since $x=-3$),

then (from there), up 4 (since $y=4$)

Put a dot there to indicate the point (-3, 4)

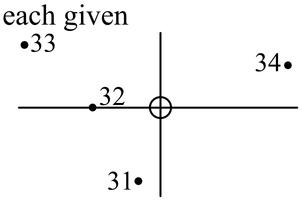


28. Join the following points in the given order: (-3, -2), (1, -4), (-3, 0), (2, 3), (-1, 2), (3, 0), (-3, -2), (-1, 2), (1, -4)



29. Two of the lines you drew cross each other. What are the coordinates of this crossing point?
30. In what quadrant does the point (*a*, *b*) lie, if $a>0$ and $b<0$?

Problems 31-34: For each given point, which of its coordinates, *x* or *y*, is larger?



D. <u>Graphing linear equations on the coordinate plane</u>:
The graph of a linear equation is a line, and one way to find the line is to join points of the line. Two points determine a line, but three are often plotted on a graph to be sure they are collinear (all in a line).

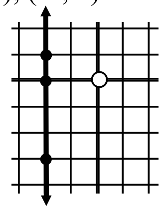
<u>Case I</u>: If the equation looks like $x=a$, then there is no restriction on *y*, so *y* can be any number. Pick 3 numbers for values of *y*, and make 3 ordered pairs so each has $x=a$. Plot and join.

example: $x = -2$

Select three y's, say -3, 0, and 1

Ordered pairs: (-2, -3), (-2, 0), (-2, 1)

Plot and join:



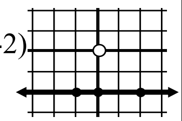
Note the slope formula gives $\frac{-3-0}{-2-(-2)} = \frac{-3}{0}$, which is not defined: a vertical line has no slope.

Case II: If the equation looks like $y = mx + b$, where either m or b (or both) can be zero, select any three numbers for values of x, and find the corresponding y values. Graph (plot) these ordered pairs and join.

example: $y = -2$

Select three x's, say -1, 0, and 2

Since y must be -2, the pairs are: (-1, -2), (0, -2), (2, -2)



The slope is $\frac{-2-(-2)}{-1-0} = \frac{0}{-1} = 0$

And the line is horizontal.

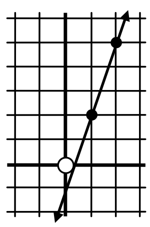
example: $y = 3x - 1$

Select 3 x's, say 0, 1, 2:

If $x = 0$, $y = 3 \bullet 0 - 1 = -1$

If $x = 1$, $y = 3 \bullet 1 - 1 = 2$

If $x = 2$, $y = 3 \bullet 2 - 1 = 5$



Ordered pairs: (0, -1), (1, 2), (2, 5)

Note the slope is $\frac{2-(-1)}{1-0} = \frac{3}{1} = 3$,

And the line is neither horizontal nor vertical.

Problems 35-41: Graph each line on the number plane and find its slope (refer to section E below if necessary):

35. $y = 3x$
36. $x - y = 3$
37. $x = 1 - y$
38. $y = 1$
39. $x = -2$
40. $y = -2x$
41. $y = \frac{1}{2}x + 1$

E. <u>Slope of a line through two points</u>:

Problems 42-47: Find the value of each of the following:

42. $\frac{3}{6} =$
43. $\frac{5-2}{1-(-1)} =$
44. $\frac{-6-(-1)}{5-10} =$
45. $\frac{0-1}{-1-4} =$
46. $\frac{0}{3} =$
47. $\frac{-2}{0} =$

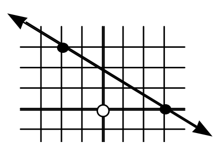
The line joining the points $P_1(x_1, y_1)$ and $P_2(x_2, y_2)$ has slope $\frac{y_2 - y_1}{x_2 - x_1}$.

example: A (3, -1), B (-2, 4)

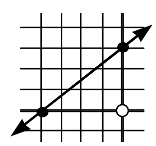
slope of $\overline{AB} = \frac{4-(-1)}{-2-3} = \frac{5}{-5} = -1$

Problems 48-52: Find the slope of the line joining the given points:

48. (-3, 1) and (-1, -4)
49. (0, 2) and (-3, -5)
50. (3, -1) and (5, -1)
51.

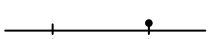


52.



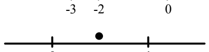
Answers:

1. D
2. E
3. C
4. F
5. B
6. G
7. B
8. Q
9. T
10. S
11. 3

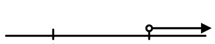


12. $-\frac{5}{2}$

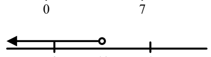
13. ½



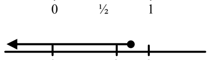
14. $x > 7$



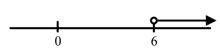
15. $x < \frac{1}{2}$



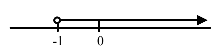
16. $x \le \frac{5}{2}$



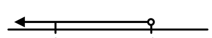
17. $x > 6$



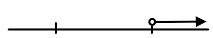
18. $x > -1$



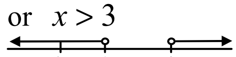
19. $x < 4$



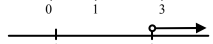
20. $x > 5$



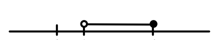
21. $x < 1$ or $x > 3$



22. $x > 2$

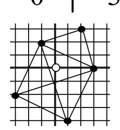


23. $1 < x \le 4$

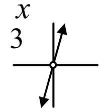


	x	y
24.	3	0
25.	-2	5
26.	5	-2
27.	0	3

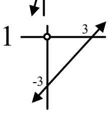
28.



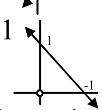
29. (0,-1)
30. IV
31. x
32. y
33. y
34. x
35. 3



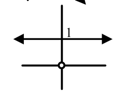
36. 1



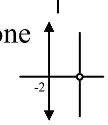
37. -1

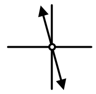


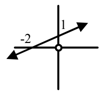
38. 0



39. none



40. -2 

41. ½ 

42. ½
43. $\frac{3}{2}$
44. 1
45. $\frac{1}{5}$
46. 0
47. none (undefined)
48. $-\frac{5}{2}$
49. $\frac{7}{3}$
50. 0
51. $-\frac{3}{5}$
52. $\frac{3}{4}$

TOPIC 6: RATIONAL EXPRESSIONS

A. <u>Simplifying fractional expressions</u>:

example: $\frac{27}{36}=\frac{9\bullet 3}{9\bullet 4}=\frac{9}{9}\bullet\frac{3}{4}=1\bullet\frac{3}{4}=\frac{3}{4}$

(note that you must be able to find a common factor—in this case 9 –in both the top and bottom in order to reduce a fraction.)

example: $\frac{3a}{12ab}=\frac{3a\bullet 1}{3a\bullet 4b}=\frac{3a}{3a}\bullet\frac{1}{4b}=1\bullet\frac{1}{4b}=\frac{1}{4b}$

(common factor: $3a$)

Problems 1-12: Reduce:

1. $\frac{13}{52}=$
2. $\frac{26}{65}=$
3. $\frac{3+6}{3+9}=$
4. $\frac{6axy}{15by}=$
5. $\frac{19a^2}{95a}=$
6. $\frac{14x-7y}{7y}=$
7. $\frac{5a+b}{5a+c}=$
8. $\frac{x-4}{4-x}=$
9. $\frac{2(x+4)(x-5)}{(x-5)(x-4)}=$
10. $\frac{x^2-9x}{x-9}=$
11. $\frac{8(x-1)^2}{6(x^2-1)}=$
12. $\frac{2x^2-x-1}{x^2-2x+1}=$

example: $\frac{3}{x}\bullet\frac{y}{15}\bullet\frac{10x}{y^2}=\frac{3\bullet 10\bullet x\bullet y}{15\bullet x\bullet y^2}=$

$\frac{3}{3}\bullet\frac{5}{5}\bullet\frac{2}{1}\bullet\frac{x}{x}\bullet\frac{y}{y}\bullet\frac{1}{y}=1\bullet 1\bullet 2\bullet 1\bullet 1\bullet\frac{1}{y}=\frac{2}{y}$

Problems 13-14: Simplify:

13. $\frac{4x}{6}\bullet\frac{xy}{y^2}\bullet\frac{3y}{2}=$
14. $\frac{x^2-3x}{x-4}\bullet\frac{x(x-4)}{2x-6}=$

B. <u>Evaluation of fractions</u>:

example: If $a=-1$ and $b=2$,
find the value of $\frac{a+3}{2b-1}$
Substitute: $\frac{-1+3}{2(2)-1}=\frac{2}{3}$

Problems 15-22: Find the value, given $a=-1$, $b=2$, $c=0$, $x=-3$, $y=1$, $z=2$:

15. $\frac{6}{b}=$
16. $\frac{x}{a}=$
17. $\frac{x}{3}=$
18. $\frac{a-y}{b}=$
19. $\frac{4x-5y}{3y-2x}=$
20. $\frac{b}{c}=$
21. $-\frac{b}{z}=$
22. $\frac{c}{z}=$

C. <u>Equivalent fractions</u>:

example: $\frac{3}{4}$ is equivalent to how many eighths?

$\frac{3}{4}=\frac{}{8}$, $\frac{3}{4}=1\bullet\frac{3}{4}=\frac{2}{2}\bullet\frac{3}{4}=\frac{2\bullet 3}{2\bullet 4}=\frac{6}{8}$

example: $\frac{6}{5a}=\frac{}{5ab}$

$\frac{6}{5a}=\frac{b}{b}\bullet\frac{6}{5a}=\frac{6b}{5ab}$

example: $\frac{3x+2}{x+1}=\frac{}{4(x+1)}$

$\frac{3x+2}{x+1}=\frac{4}{4}\bullet\frac{3x+2}{x+1}=\frac{12x+8}{4x+4}$

example: $\frac{x-1}{x+1}=\frac{}{(x+1)(x-2)}$

$\frac{x-1}{x+1}=\frac{(x-2)(x-1)}{(x-2)(x+1)}=\frac{x^2-3x+2}{(x+1)(x-2)}$

Problems 23-27: Complete:

23. $\frac{4}{9}=\frac{}{72}$
24. $\frac{3x}{7}=\frac{}{7y}$
25. $\frac{x+3}{x+2}=\frac{}{(x-1)(x+2)}$
26. $\frac{30-15a}{15-15b}=\frac{}{(1+b)(1-b)}$
27. $\frac{x-6}{6-x}=\frac{}{-2}$

How to get the lowest common denominator (LCD) by finding the least common multiple (LCM) of all denominators:

example: $\frac{5}{6}$ and $\frac{8}{15}$.

First find LCM of 6 and 15:

$6=2\bullet 3$
$15=3\bullet 5$
$\text{LCM}=2\bullet 3\bullet 5=30$, so
$\frac{6}{5}=\frac{25}{30}$, and $\frac{8}{15}=\frac{16}{30}$

example: $\frac{3}{4}$ and $\frac{1}{6a}$:

$4=2\bullet 2$
$6a=2\bullet 3\bullet a$
$\text{LCM}=2\bullet 2\bullet 3\bullet a=12a$, so
$\frac{3}{4}=\frac{9a}{12a}$, and $\frac{1}{6a}=\frac{2}{12a}$

example: $\frac{3}{x+2}$ and $\frac{-1}{x-2}$

$\text{LCM}=(x+2)(x-2)$, so
$\frac{3}{x+2}=\frac{3\bullet(x-2)}{(x+2)(x-2)}$, and $\frac{-1}{x-2}=\frac{-1\bullet(x+2)}{(x+2)(x-2)}$

Problems 28-33: Find equivalent fractions with the lowest common denominator:

28. $\frac{2}{3}$ and $\frac{2}{9}$
29. $\frac{3}{x}$ and 5
30. $\frac{x}{3}$ and $\frac{-4}{x+1}$
31. $\frac{3}{x-2}$ and $\frac{4}{2-x}$
32. $\frac{-4}{x-3}$ and $\frac{5}{x+3}$
33. $\frac{1}{x}$ and $\frac{3x}{x+1}$

D. <u>Adding and subtracting fractions</u>:

If denominators are the same, combine the numbers:

example: $\frac{3x}{y}-\frac{x}{y}=\frac{3x-x}{y}=\frac{2x}{y}$

Problems 34-38: Find the sum or difference as indicated (reduce if possible):

34. $\frac{4}{7}+\frac{2}{7}=$
35. $\frac{3}{x-3}-\frac{x}{x-3}=$
36. $\frac{b-a}{b+a}-\frac{a-b}{b+a}=$
37. $\frac{x+2}{x^2+2x}-\frac{3y^2}{xy^2}=$
38. $\frac{3a}{b}+\frac{2}{b}-\frac{a}{b}=$

If denominators are different, find equivalent fractions with common denominators, then proceed as before (combine numerators):

example: $\frac{a}{2}-\frac{a}{4}=\frac{2a}{4}-\frac{a}{4}=\frac{2a-a}{4}=\frac{a}{4}$

example: $\frac{3}{x-1}+\frac{1}{x+2}$

$$=\frac{3(x+2)}{(x-1)(x+2)}+\frac{(x-1)}{(x-1)(x+2)}$$

$$=\frac{3x+6+x-1}{(x-1)(x+2)}=\frac{4x+5}{(x-1)(x+2)}$$

Problems 39-51: Find the sum or difference:

39. $\frac{3}{a}-\frac{1}{2a}=$
40. $\frac{3}{x}-\frac{2}{a}=$
41. $\frac{4}{5}-\frac{2}{x}=$
42. $\frac{2}{5}+2=$
43. $\frac{a}{b}-2=$
44. $a-\frac{c}{b}=$
45. $\frac{1}{a}+\frac{1}{b}=$
46. $a-\frac{1}{a}=$
47. $\frac{x}{x-1}+\frac{x}{1-x}=$
48. $\frac{3x-2}{x-2}-\frac{2}{x+2}=$
49. $\frac{2x-1}{x+1}-\frac{2x-1}{x-2}=$
50. $\frac{x}{x-2}-\frac{4}{x^2-2x}=$
51. $\frac{x}{x-2}-\frac{4}{x^2-4}=$

E. <u>Multiplying fractions</u>:

Multiply the tops, multiply the bottoms, reduce if possible:

example: $\frac{3}{4}\bullet\frac{2}{5}=\frac{6}{20}=\frac{3}{10}$

example: $\frac{3(x+1)}{x-2}\bullet\frac{x^2-4}{x^2-1}$

$$=\frac{3(x+1)(x+2)(x-2)}{(x-2)(x+1)(x-1)}=\frac{3x+6}{x-1}$$

52. $\frac{2}{3}\bullet\frac{3}{8}=$
53. $\frac{a}{b}\bullet\frac{c}{d}=$
54. $\frac{2}{7a}\bullet\frac{ab}{12}=$
55. $\left(\frac{3}{4}\right)^2=$
56. $\left(2\frac{1}{2}\right)^2=$
57. $\left(\frac{2a^3}{5b}\right)^3=$
58. $\frac{3(x+4)}{5y}\bullet\frac{5y^3}{x^2-16}=$
59. $\frac{x+3}{3x}\bullet\frac{x^2}{2x+6}=$

F. <u>Dividing fractions</u>:

A nice way to do this is to make a compound fraction and then multiply the top and bottom (of the big fraction) by the LCD of both:

example: $\frac{a}{b}\div\frac{c}{d}=\frac{\frac{a}{b}}{\frac{c}{d}}=\frac{\frac{a}{b}\bullet bd}{\frac{c}{d}\bullet bd}=\frac{ad}{bc}$

example: $\frac{7}{\frac{2}{3}-\frac{1}{2}}=\frac{7\bullet 6}{\left(\frac{2}{3}-\frac{1}{2}\right)\bullet 6}=\frac{42}{4-3}=42/1=42$

example: $\frac{5x}{2y}\div 2x=\frac{\frac{5x}{2y}}{2x}=\frac{\frac{5x}{2y}\bullet 2y}{2x\bullet 2y}=\frac{5x}{4xy}=\frac{5}{4y}$

60. $\frac{\frac{3}{4}}{\frac{2}{3}}=$
61. $11\frac{3}{8}\div\frac{3}{4}=$
62. $\frac{3}{4}\div 2=$
63. $\frac{a}{b}\div 3=$
64. $\frac{3}{a}\div\frac{b}{3}=$
65. $\frac{2a-b}{\frac{1}{2}}=$
66. $\frac{a-4}{\frac{3}{a}-2}=$
67. $\frac{\frac{x+7}{x^2-9}}{\frac{1}{x-3}}=$
68. $\frac{2}{\frac{3}{4}}=$
69. $\frac{\frac{2}{3}}{4}=$
70. $\frac{\frac{a}{b}}{c}=$
71. $\frac{a}{\frac{b}{c}}=$

<u>Answers</u>:

1. $1/4$
2. $2/5$
3. $3/4$
4. $\frac{2ax}{5b}$
5. $a/5$
6. $\frac{2x-y}{y}$
7. $\frac{5a+b}{5a+c}$
8. -1
9. $\frac{2(x+4)}{x-4}$
10. x
11. $\frac{4(x-1)}{3(x+1)}$
12. $\frac{2x+1}{x-1}$
13. x^2
14. $x^2/2$
15. 3
16. 3
17. -1
18. -1
19. $-17/9$
20. undefined $\frac{2}{0}$
21. -1
22. 0
23. 32
24. $3xy$
25. x^2+2x-3 or $(x-1)(x+3)$
26. $2+2b-a-ab$ or $(1+b)(2-a)$
27. 2
28. $\frac{6}{9}$, $\frac{2}{9}$
29. $\frac{3}{x}$, $\frac{5x}{x}$
30. $\frac{x(x+1)}{3(x+1)}$, $\frac{-12}{3(x+1)}$
31. $\frac{3}{x-2}$, $\frac{-4}{x-2}$

32. $\frac{-4(x+3)}{(x-3)(x+3)}, \frac{-5(x-3)}{(x-3)(x+3)}$
33. $\frac{x+1}{x(x+1)}, \frac{3x^2}{x(x+1)}$
34. $6/7$
35. -1
36. $\frac{2b-2a}{b+a}$
37. $-2/x$
38. $\frac{2a+2}{b}$
39. $\frac{5}{2a}$
40. $\frac{3a-2x}{ax}$
41. $\frac{4x-10}{5x}$
42. $12/5$
43. $\frac{a-2b}{b}$
44. $\frac{ab-c}{b}$
45. $\frac{a+b}{ab}$
46. $\frac{a^2-1}{a}$
47. 0
48. $\frac{3x^2+2x}{x^2-4}$
49. $\frac{-3(2x-1)}{(x+1)(x-2)}$
50. $\frac{x+2}{x}$
51. $\frac{x^2+2x-4}{x^2-4}$
52. $1/4$
53. $\frac{ac}{bd}$
54. $\frac{b}{42}$
55. $\frac{9}{16}$
56. $\frac{25}{4}$
57. $\frac{8a^9}{125b^3}$
58. $\frac{3y^2}{x-4}$
59. $x/6$
60. $9/8$
61. $91/6$
62. $3/8$
63. $\frac{a}{3b}$
64. $\frac{9}{ab}$
65. $4a-2b$
66. $\frac{a^2-4a}{3-2a}$
67. $\frac{x+7}{x+3}$
68. $8/3$
69. $1/6$
70. $\frac{a}{bc}$
71. $\frac{ac}{b}$

TOPIC 7: EXPONENTS and SQUARE ROOT

A. <u>Positive integer exponents</u>:

a^b means use *a* as a factor *b* times. (*b* is the exponent or power of *a*.)

example: 2^5 means $2\bullet 2\bullet 2\bullet 2\bullet 2$, and has value 32.
example: $c\bullet c\bullet c = c^3$

Problems 1-14: Find the value:

1. $2^3 =$
2. $3^2 =$
3. $-4^2 =$
4. $(-4)^2 =$
5. $0^4 =$
6. $1^4 =$
7. $\left(\frac{2}{3}\right)^4 =$
8. $(.2)^2 =$
9. $\left(1\frac{1}{2}\right)^2 =$
10. $2^{10} =$
11. $(-2)^9 =$
12. $\left(2\frac{2}{3}\right)^2 =$
13. $(-1.1)^3 =$
14. $3^2\bullet 2^3 =$

example: Simplify:
$a\bullet a\bullet a\bullet a\bullet a = a^5$

Problems 15-18: Simplify:

15. $3^2\bullet x^4 =$
16. $2^4\bullet b\bullet b\bullet b =$
17. $4^2(-x)(-x)(-x) =$
18. $(-y)^4 =$

B. <u>Integer exponents</u>:

I. $a^b\bullet a^c = a^{b+c}$
II. $\frac{a^b}{a^c} = a^{b-c}$
III. $\left(a^b\right)^c = a^{bc}$
IV. $(ab)^c = a^c\bullet b^c$
V. $\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$
VI. $a^0 = 1$ (if $a \neq 0$)
VII. $a^{-b} = \frac{1}{a^b}$

Problems 19-28: Find *x*:

19. $2^3\bullet 2^4 = 2^x$
20. $\frac{2^3}{2^4} = 2^x$
21. $3^{-4} = \frac{1}{3^x}$
22. $\frac{5^2}{5^2} = 5^x$
23. $\left(2^3\right)^4 = 2^x$
24. $8 = 2^x$
25. $a^3\bullet a = a^x$
26. $\frac{b^{10}}{b^5} = b^x$
27. $\frac{1}{c^{-4}} = c^x$
28. $\frac{a^{3y-2}}{a^{2y-3}} = a^x$

Problems 29-41: Find the value:

29. $7x^0 =$
30. $3^{-4} =$
31. $2^3\bullet 2^4 =$
32. $0^5 =$
33. $5^0 =$
34. $(-3)^3 - 3^3 =$
35. $x^{c+3}\bullet x^{c-3} =$
36. $\frac{x^{c+3}}{x^{c-3}} =$
37. $\frac{2x^{-3}}{6x^{-4}} =$
38. $\left(a^{x+3}\right)^{x-3} =$
39. $\left(x^3\right)^2 =$
40. $\left(3x^3\right)^2 =$
41. $\left(-2xy^2\right)^3 =$

C. <u>Scientific notation</u>:

example: $32800 = 3.2800\times 10^4$ if the zeros in the ten's and one's places are significant. If the one's zero is not, write 3.280×10^4; if neither is significant: 3.28×10^4.
example: $.004031 = 4.031\times 10^{-3}$

example: $2\times10^2=200$
example: $9.9\times10^{-1}=.99$

Note that scientific form always looks like $a\times10^n$ where $1\le a<10$, and *n* is an integer power of 10.

Problems 42-45: Write in scientific notation:

42. $93{,}000{,}000=$
43. $.000042=$
44. $5.07=$
45. $-32=$

Problems 46-48: Write in standard notation:

46. $1.4030\times10^3=$
47. -9.11×10^{-2}
48. $4\times10^{-6}=$

To compute with numbers written in scientific form, separate the parts, compute, then recombine.

example: $(3.14\times10^5)(2)=$
$(3.14)(2)\times10^5=6.28\times10^5$
example: $\frac{4.28\times10^6}{2.14\times10^{-2}}=$
$\frac{4.28}{2.14}\times\frac{10^6}{10^{-2}}=2.00\times10^8$
example: $\frac{2.01\times10^{-3}}{8.04\times10^{-6}}=$
$.250\times10^3=2.50\times10^2$

Problems 49-56: Write answer in scientific notation:

49. $10^{40}\times10^{-2}=$
50. $\frac{10^{-40}}{10^{-10}}=$
51. $\frac{1.86\times10^4}{3\times10^{-1}}=$
52. $\frac{3.6\times10^{-5}}{1.8\times10^{-8}}=$
53. $\frac{1.8\times10^{-8}}{3.6\times10^{-5}}=$
54. $(4\times10^{-3})^2=$
55. $(2.5\times10^2)^{-1}=$
56. $\frac{(-2.92\times10^3)(4.1\times10^7)}{-8.2\times10^{-3}}=$

D. <u>Simplification of square roots</u>:

$\sqrt{ab}=\sqrt{a}\bullet\sqrt{b}$ if *a* and *b* are both non-negative ($a\ge0$ and $b\ge0$).

example: $\sqrt{32}=\sqrt{16}\bullet\sqrt{2}=4\sqrt{2}$
example: $\sqrt{75}=\sqrt{3}\bullet\sqrt{25}=\sqrt{3}\bullet5=5\sqrt{3}$
example: If $x\ge0$, $\sqrt{x^6}=x^3$
If $x<0$, $\sqrt{x^6}=|x^3|$

Note: $\sqrt{a}=b$ means (by definition) that
1) $b^2=a$, and
2) $b\ge0$

Problems 57-69: Simplify (assume all square roots are real numbers):

57. $\sqrt{81}=$
58. $-\sqrt{81}=$
59. $2\sqrt{9}=$
60. $4\sqrt{9}=$
61. $\sqrt{40}=$
62. $3\sqrt{12}=$
63. $\sqrt{52}=$
64. $\sqrt{\frac{9}{16}}=$
65. $\sqrt{.09}=$
66. $\sqrt{x^5}=$
67. $\sqrt{4x^6}=$
68. $\sqrt{a^2}=$
69. $\sqrt{a^3}=$

E. <u>Adding and subtracting square roots</u>:

example: $\sqrt{5}+2\sqrt{5}=3\sqrt{5}$
example: $\sqrt{32}-\sqrt{2}=4\sqrt{2}-\sqrt{2}=3\sqrt{2}$

Problems 70-73: Simplify:

70. $\sqrt{5}+\sqrt{5}=$
71. $2\sqrt{3}+\sqrt{27}-\sqrt{75}=$
72. $3\sqrt{2}+\sqrt{2}=$
73. $5\sqrt{3}-\sqrt{3}=$

F. <u>Multiplying square roots</u>:

$\sqrt{a}\bullet\sqrt{b}=\sqrt{ab}$ if $a\ge0$ and $b\ge0$.

example: $\sqrt{6}\bullet\sqrt{24}=\sqrt{6\bullet24}=\sqrt{144}=12$
example: $\sqrt{2}\bullet\sqrt{6}=\sqrt{12}=\sqrt{4}\bullet\sqrt{3}=2\sqrt{3}$
example: $(5\sqrt{2})(3\sqrt{2})=15\sqrt{4}=15\bullet2=30$

Problems 74-79: Simplify:

74. $\sqrt{3}\bullet\sqrt{3}=$
75. $\sqrt{3}\bullet\sqrt{4}=$
76. $(2\sqrt{3})(3\sqrt{2})=$
77. $(\sqrt{9})^2=$
78. $(\sqrt{5})^2=$
79. $(\sqrt{3})^4=$

Problems 80-81: Find the value of *x*:

80. $\sqrt{4}\bullet\sqrt{9}=\sqrt{x}$
81. $3\sqrt{2}\bullet\sqrt{5}=3\sqrt{x}$

G. <u>Dividing square roots</u>:

$\sqrt{a}\div\sqrt{b}=\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$, if $a\ge0$ and $b>0$.

example: $\sqrt{2}\div\sqrt{64}=\frac{\sqrt{2}}{\sqrt{64}}=\frac{\sqrt{2}}{8}$ (or $\frac{1}{8}\sqrt{2}$)

Problems 82-86: Simplify:

82. $\sqrt{3}\div\sqrt{4}=$
83. $\frac{\sqrt{9}}{\sqrt{25}}=$
84. $\frac{\sqrt{49}}{2}=$
85. $\sqrt{36}\div4=$
86. $\frac{-8}{\sqrt{16}}=$

If a fraction has a square root on the bottom, it is sometimes desirable to find an equivalent fraction with no root on the bottom. This is called rationalizing the denominator.

example: $\sqrt{\frac{5}{8}}=\frac{\sqrt{5}}{\sqrt{8}}=\frac{\sqrt{5}}{\sqrt{8}}\bullet\frac{\sqrt{2}}{\sqrt{2}}=\frac{\sqrt{10}}{\sqrt{16}}=\frac{\sqrt{10}}{4}$
example: $\frac{1}{\sqrt{3}}=\frac{1}{\sqrt{3}}\bullet\frac{\sqrt{3}}{\sqrt{3}}=\frac{\sqrt{3}}{\sqrt{9}}=\frac{\sqrt{3}}{3}$

Problems 87-94: Simplify:

87. $\sqrt{\frac{9}{4}} =$

88. $\frac{\sqrt{18}}{\sqrt{9}} =$

89. $\frac{\sqrt{4}}{9} =$

90. $\sqrt{\frac{3}{2}} =$

91. $\frac{1}{\sqrt{5}} =$

92. $\frac{3}{\sqrt{3}} =$

93. $\frac{\sqrt{a}}{\sqrt{b}} =$

94. $\sqrt{2} + \frac{1}{\sqrt{2}} =$

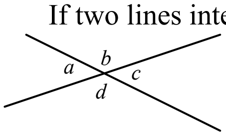
<u>Answers</u>:

1. 8
2. 9
3. -16
4. 16
5. 0
6. 1
7. $^{16}/_{81}$
8. .04
9. $^{9}/_{4}$
10. 1024
11. -512
12. $^{64}/_{9}$
13. -1.331
14. 72
15. $9x^4$
16. $16b^3$
17. $-16x^3$
18. y^4
19. 7
20. -1
21. 4
22. 0
23. 12
24. 3
25. 4
26. 5
27. 4
28. $y+1$
29. 7
30. $^{1}/_{81}$
31. 128
32. 0
33. 1
34. -54
35. x^{2c}
36. x^6
37. $^{x}/_{3}$
38. a^{x^2-9}
39. x^6
40. $9x^6$
41. $-8x^3y^6$
42. 9.3×10^7
43. 4.2×10^{-5}
44. 5.07
45. -3.2×10
46. 1403.0
47. -.0911
48. .000004
49. 1×10^{38}
50. 1×10^{-30}
51. 6.2×10^4
52. 2.0×10^3
53. 5.0×10^{-4}
54. 1.6×10^{-5}
55. 4.0×10^{-3}
56. 1.46×10^{13}
57. 9
58. -9
59. 6
60. 12
61. $2\sqrt{10}$
62. $6\sqrt{3}$
63. $2\sqrt{13}$
64. $^{3}/_{4}$
65. .3
66. $x^2\sqrt{x}$
67. $2\left|x^3\right|$
68. $|a|$
69. $a\sqrt{a}$
70. $2\sqrt{5}$
71. 0
72. $4\sqrt{2}$
73. $4\sqrt{3}$
74. 3
75. $2\sqrt{3}$
76. $6\sqrt{6}$
77. 9
78. 5
79. 9
80. 36
81. 10
82. $^{\sqrt{3}}/_{2}$
83. $^{3}/_{5}$
84. $^{7}/_{2}$
85. $^{3}/_{2}$
86. -2
87. $^{3}/_{2}$
88. $\sqrt{2}$
89. $^{2}/_{9}$
90. $^{\sqrt{6}}/_{2}$
91. $^{\sqrt{5}}/_{5}$
92. $\sqrt{3}$
93. $^{\sqrt{ab}}/_{b}$
94. $\frac{3\sqrt{2}}{2}$

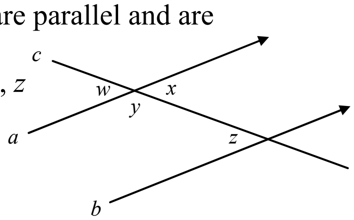
TOPIC 8: GEOMETRIC MEASUREMENT

A. <u>Intersecting lines and parallels</u>:

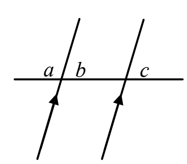
If two lines intersect as shown, adjacent angles add to 180°. For example, $a+d=180°$. Non-adjacent angles are equal: for example, $a=c$.



If two lines, *a* and *b*, are parallel and are cut by a third line *c*, forming angles *w*, *x*, *y*, *z* as shown, then $x=z$, $w+y=180°$, so $z+y=180°$.

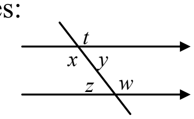


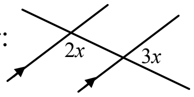
example: If $a=3x$ and $c=x$, find the measure of *c*.
$b=c$, so $b=x$.
$a+b=180$, so $3x+x=180$, giving $4x=180$, or $x=45$
Thus $c=x=45°$



Problems 1-4: Given $x=127°$, find the measures of the other angles:

1. *t*
2. *y*
3. *z*
4. *w*

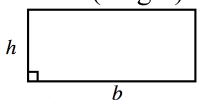


5. Find x: 

B. <u>Formulas for perimeter *P* and area *A* of triangles, squares, rectangles, and parallelograms</u>:

<u>Rectangle</u>, base b, altitude (height) h:

$P = 2b + 2h$
$A = bh$

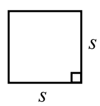


If a wire is bent in the shape, the perimeter is the length of the wire, and the area is the number of square units enclosed by the wire.

> *example:* Rectangle with $b = 7$ and $h = 8$:
> $P = 2b + 2h = 2 \bullet 7 + 2 \bullet 8 =$
> $14 + 16 = 30$ units
> $A = bh = 7 \bullet 8 = 56$ sq. units

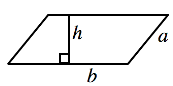
A <u>square</u> is a rectangle with all sides equal, so the formulas are the same (and simpler if the side length is s):

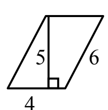
$P = 4s$
$A = s^2$



> *example:* Square with side $11cm$ has
> $P = 4s = 4 \bullet 11 = 44cm$
> $A = s^2 = 11^2 = 121cm^2$ (sq. cm)

A <u>parallelogram</u> with base b and height h has $A = bh$. If the other side length is a, then $P = 2a + 2b$.

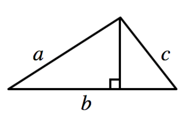


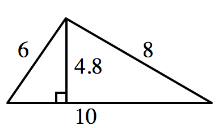
> *example:* Parallelogram has sides 4 and 6, and 5 is the length of the altitude perpendicular to the side 4.
> 
>
> $P = 2a + 2b = 2 \bullet 6 + 2 \bullet 4 = 12 + 8 = 20$ units
> $A = bh = 4 \bullet 5 = 20$ sq. units

In a <u>triangle</u> with side lengths a, b, c and h is the altitude to side b,

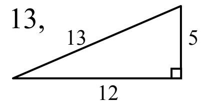
$P = a + b + c$

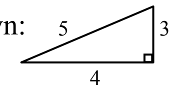
$A = \frac{1}{2}bh = \frac{bh}{2}$



> *example:*
> 
>
> $P = a + b + c$
> $= 6 + 8 + 10$
> $= 24$ units
> $A = \frac{1}{2}bh = \frac{1}{2}(10)(4.8) = 24$ sq. units

Problems 6-13: Find P and A for each of the following figures:

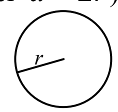
6. Rectangle with sides 5 and 10.
7. Rectangle, sides 1.5 and 4.
8. Square with side 3 mi.
9. Square, side $\frac{3}{4}$ yd.
10. Parallelogram with sides 36 and 24, and height 10 (on side 36).
11. Parallelogram, all sides 12, altitude 6.
12. Triangle with sides 5, 12, 13, and 5 is the height on side 12. 

13. The triangle shown: 

C. <u>Formulas for circle area *A* and circumference *C*</u>:

A circle with radius r (and diameter $d = 2r$) has distance around (circumference)

$C = \pi d$ or $C = 2\pi r$



(If a piece of wire is bent into a circular shape, the circumference is the length of wire.)

> *example:* A circle with radius $r = 70$ has
> $d = 2r = 170$ and <u>exact</u> circumference
> $C = 2\pi r = 2 \bullet \pi \bullet 70 = 140\pi$ units.
> If π is approximated by $\frac{22}{7}$,
> $C = 140\pi = 140\left(\frac{22}{7}\right) = 440$ units <u>approximately</u>.
> If π is approximated by 3.1, the <u>approximate</u>
> $C = 140(3.1) = 434$ units.

The area of a circle is $A = \pi r^2$:

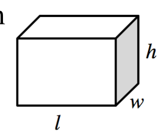
> *example:* If $r = 8$
> $A = \pi r^2 = \pi \bullet 8^2 = 64\pi$ sq. units

Problems 14-16: Find C and A for each circle:

14. $r = 5$ units
15. $r = 10$ feet
16. $d = 4$ km

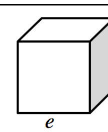
D. <u>Formulas for volume *V*</u>:

A <u>rectangular solid</u> (box) with length l, width w, and height h, has volume $V = lwh$.



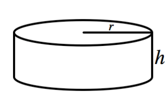
> *example:* A box with dimensions 3, 7, and 11 has what volume?
> $V = lwh = 3 \bullet 7 \bullet 11 = 231$ cu. units

A <u>cube</u> is a box with all edges equal. If the edge is e the volume $V = e^3$



> *example:* A cube has edge 4 cm.
> $V = e^3 = 4^3 = 64cm^3$ (cu. cm)

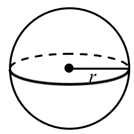
A (right circular) <u>cylinder</u> with radius r and altitude h has $V = \pi r^2 h$



> *example:* A cylinder has $r = 10$ and $h = 14$.

The exact volume is
$V = \pi r^2 h = \pi \bullet 10^2 \bullet 14 = 1400\pi$ cu. units
If π is approximated by $\frac{22}{7}$,
$V = 1400 \bullet \frac{22}{7} = 4400$ cu. units
If π is approximated by 3.14,
$V = 1400(3.14) = 4396$ cu. units

A sphere (ball) with radius r has volume $V = \frac{4}{3}\pi r^3$



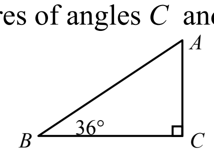
example: The exact volume of a sphere with radius 6 *in.* is $V = \frac{4}{3}\pi r^3 = \frac{4}{3} \bullet \pi \bullet 6^3$
$= \frac{4}{3}\pi(216) = 288\pi\ in^3$

Problems 17-24: Find the exact volume of each of the following solids:

17. Box, 6 by 8 by 9.
18. Box, $1\frac{2}{3}$ by $\frac{5}{6}$ by $2\frac{2}{5}$.
19. Cube with edge 10.
20. Cube, edge .5 .
21. Cylinder with $r = 5$, $h = 10$
22. Cylinder, $r = \sqrt{3}$, $h = 2$
23. Sphere with radius $r = 2$.
24. Sphere with radius $r = \frac{3}{4}$.

E. Sum of the interior angles of a triangle:
The three angles of any triangle add to $180°$.

example: Find the measures of angles C and A:
$\angle C$ (angle C) is marked to show its measure is $90°$.



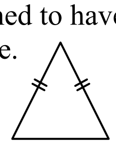
$\angle B + \angle C = 36 + 90 = 126$, so
$\angle A = 180 - 126 = 54°$

Problems 25-29: Given two angles of a triangle, find the measure of the third angle:

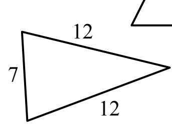
25. $30°$, $60°$
26. $115°$, $36°$
27. $90°$, $17°$
28. $82°$, $82°$
29. $68°$, $44°$

F. Isosceles triangles:

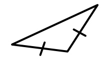
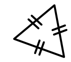
An isosceles triangle is defined to have at least two sides with equal measure.
The equal sides may be marked:



Or the measures may be given:

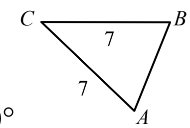


Problems 30-35: Is the triangle isosceles?

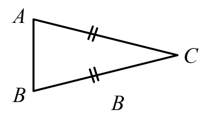
30. Sides 3, 4, 5
31. Sides 7, 4, 7
32. Sides 8, 8, 8
33. 
34. 
35. 

The angles which are opposite the equal sides also have equal measures (and all three angles add to $180°$).

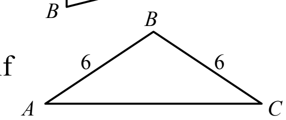
example: Find the measures of $\angle A$ and $\angle C$, given $\angle B = 65°$:
$\angle A + \angle B + \angle C = 180$, and
$\angle A = \angle B = 65$, so $\angle C = 50°$



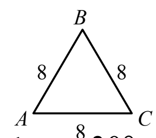
36. Find measures of $\angle A$ and $\angle B$, if $\angle C = 30°$.



37. Find measures of $\angle B$ and $\angle C$, if $\angle A = 30°$.



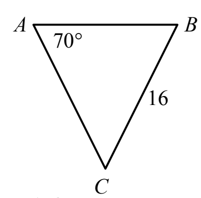
38. Find measure of $\angle A$.



39. If the angles of a triangle are $30°$, $60°$, and $90°$, can it be isosceles?
40. If two angles of a triangle are $45°$ and $60°$, can it be isosceles?

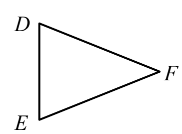
If a triangle has equal angles, the sides opposite these angles also have equal measures.

example: Find the measures of $\angle B$, $\overline{AB}$ and $\overline{AC}$, given this figure, and $\angle C = 40°$:



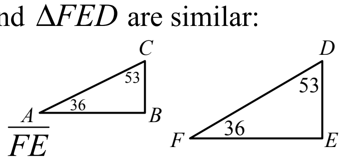
$\angle B = 70°$(because all angles add to $180°$)
Since $\angle A = \angle B$, $AC = BC = 16$.
AB can be found with trig -- later.

41. Can a triangle be isosceles and have a $90°$?
42. Given $\angle D = \angle E = 68°$ and $DF = 6$. Find the measure of $\angle F$ and length of $\overline{FE}$:



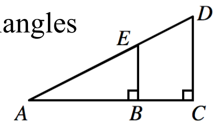
G. Similar triangles:
If two angles of one triangle are equal to two angles of another triangle, then the triangles are similar.

example: ΔABC and ΔFED are similar:



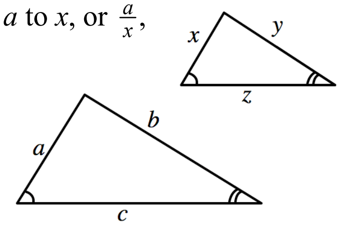
The pairs of corresponding sides are $\overline{AB}$ and $\overline{FE}$
$\overline{BC}$ and $\overline{ED}$, and $\overline{AC}$ and $\overline{FD}$.

43. Name two similar triangles and list the pairs of corresponding sides.



If two triangles are similar, any two corresponding sides have the same ratio (fraction value):

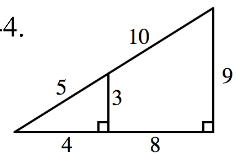
example: the ratio a to x, or $\frac{a}{x}$, is the same as $\frac{b}{y}$ and $\frac{c}{z}$. Thus, $\frac{a}{x} = \frac{b}{y}$, $\frac{a}{x} = \frac{c}{z}$, and $\frac{b}{y} = \frac{c}{z}$.



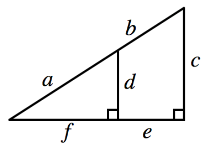
Each of these equations is called a proportion

Problems 44-45: Write proportions for the two similar triangles:

44.

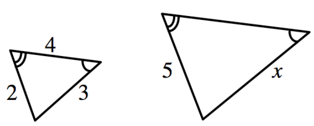


45.



example: Find x:

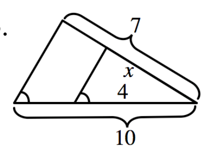
Write and solve a proportion:



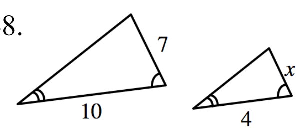
$\frac{2}{5} = \frac{3}{x}$, so $2x = 15$, $x = 7\frac{1}{2}$

Problems 46-49: Find x:

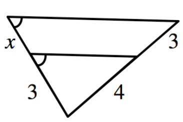
46.



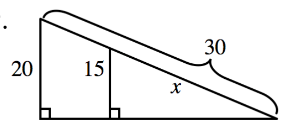
48.



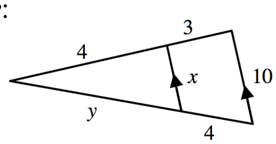
47.



49.

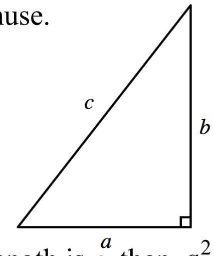


50. Find x and y:



H. Pythagorean theorem:

In any triangle with a $90°$ (right) angle, the sum of the squares of the legs equals the square of the hypotenuse. (The legs are the two shorter sides; the hypotenuse is the longest side.) If the legs have lengths a and b,



and the hypotenuse length is c, then $a^2 + b^2 = c^2$ (In words, 'In a right triangle, leg squared plus leg squared equals hypotenuse squared.')

example: A right triangle has hypotenuse 5 and one leg 3. Find the other leg.

Since $leg^2 + leg^2 = hyp^2$,

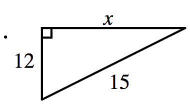
$$3^2 + x^2 = 5^2$$
$$9 + x^2 = 25$$
$$x^2 = 25 - 9 = 16$$
$$x = \sqrt{16} = 4$$

Problems 51-54: Each line of the chart lists two sides of a right triangle. Find the length of the third side:

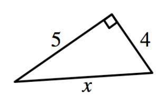
	leg	leg	hypotenuse
51.		15	17
52.	8		10
53.	5	12	
54.	$\sqrt{2}$	$\sqrt{3}$	

Problems 55-56: Find x:

55.



56.



If the sum of the squares of two sides of a triangle is the same as the square of the third side, the triangle is a right triangle.

example: Is a triangle with sides 20, 29, 21 a right triangle?

$20^2 + 21^2 = 29^2$, so it is a right triangle.

Problems 57-59: Is a triangle right, if it has sides:

57. 17, 8, 15
58. 4, 5, 6
59. 60, 61, 11

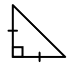
Answers:

1. 127°
2. 53°
3. 53°
4. 127°
5. 36°

	P	A
6.	30 un	50 un^2
7.	11 un	6 un^2
8.	12 mi	9 mi^2
9.	3 yd	$\frac{9}{16}$ yd^2
10.	120 un	360 un^2
11.	48 un	72 un^2
12.	30 un	30 un^2
13.	12 un	6 un^2

	C	A
14.	10πun	25πun^2
15.	20πft	100πft^2
16.	4πkm	4πkm^2

17. 432
18. $10/3$
19. 1000
20. .125

21. 250π
22. 6π
23. $32\pi/3$
24. $9\pi/16$
25. 90°
26. 29°
27. 73°
28. 16°
29. 68°
30. no
31. yes
32. yes
33. yes
34. yes
35. can't tell
36. 75° each
37. 120°, 30°
38. 60°
39. no
40. no
41. yes: 
42. 44°, 6
43. ΔABE, ΔACD
    AB, AC
    AE, AD
    BE, CD
44. $\frac{3}{9}=\frac{5}{15}=\frac{4}{12}$
45. $\frac{d}{c}=\frac{a}{a+b}=\frac{f}{f+e}$
46. $14/5$
47. $9/4$
48. $14/5$
49. $45/2$
50. $40/7$, $16/3$
51. 8
52. 6
53. 13
54. $\sqrt{5}$
55. 9
56. $\sqrt{41}$
57. yes
58. no
59. yes

TOPIC 9: WORD PROBLEMS

A. <u>Arithmetic, percent, and average</u>:

1. What is the number, which when multiplied by 32, gives $32 \bullet 46$?
2. If you square a certain number, you get 9^2. What is the number?
3. What is the power or 36 that gives 36^2?
4. Find 3% of 36.
5. 55 is what percent of 88?
6. What percent of 55 is 88?
7. 45 is 80% of what number?
8. What is 8.3% of $7000?
9. If you get 36 on a 40-question test, what percent is this?
10. The 3200 people who vote in an election are 40% of the people registered to vote. How many are registered?

Problems 11-13: Your wage is increased by 20%, then the new amount is cut by 20% (of the new amount):

11. Will this result in a wage which is higher than, lower than, or the same as the original wage?
12. What percent of the original wage is this final wage?
13. If the above steps were reversed (20% cut followed by 20% increase), the final wage would be what percent of the original wage?

Problems 14-16: If A is increased by 25%, it equals B:

14. Which is larger, B or the original A?
15. B is what percent of A?
16. A is what percent of B?
17. What is the average of 87, 36, 48, 59, and 95?
18. If two test scores are 85 and 60, what minimum score on the next test would be needed for an overall average of 80?
19. The average height of 49 people is 68 inches. What is the new average height if a 78-inch person joins the group?

B. <u>Algebraic substitution and evaluation</u>:

Problems 20-24: A certain TV uses 75 watts of power, and operates on 120 volts:

20. Find how many amps of current it uses, from the relationship: volts times amps equals watts.
21. 1000 watts = 1 kilowatt (*kw*). How many kilowatts does the TV use?
22. *Kw* times hours = kilowatt-hours (*kwh*). If the TV is on for six hours a day, how many *kwh* of electricity are used?
23. If the set is on for six hours every day of a 30-day month, how many *kwh* are used for the month?
24. If the electric company charges 8¢ per *kwh*, what amount of the month's bill is for TV power?

Problems 25-33: A plane has a certain speed in still air, where it goes 1350 miles in three hours:

25. What is its (still air) speed?
26. How far does the plane go in 5 hours?
27. How far does it go in x hours?
28. How long does it take to fly 2000 miles?
29. How long does it take to fly y miles?
30. If the plane flies against a 50 *mph* headwind, what is its ground speed?

31. If the plane flies against a headwind of z *mph*, what is its ground speed?
32. If it has fuel for 7.5 hours of flying time, how far can it go against the headwind of 50 *mph*?
33. If the plane has fuel for t hours of flying time, how far can it go against the headwind of z *mph*?

C. <u>Ratio and proportion</u>:

Problems 34-35: x is to y as 3 is to 5:

34. Find y when x is 7.
35. Find x when y is 7.

Problems 36-37: s is proportional to P, and $P = 56$ when $s = 14$:

36. Find s when $P = 144$.
37. Find P when $s = 144$.

Problems 38-39: Given $3x = 4y$:

38. Write the ratio $x : y$ as the ratio of two integers
39. If $x = 3$, find y.

Problems 40-41: x and y are numbers, and two x's equal three y's:

40. Which of x or y must be larger?
41. What is the ratio of x to y?

Problems 42-44: Half of x is the same as one-third of y:

42. Which of x and y is the larger?
43. Write the ratio $x : y$ as the ratio of two integers.
44. How many x's equal 30 y's?

D. <u>Problems leading to one linear equation</u>:

45. 36 is three-fourths of what number?
46. What number is $\frac{3}{4}$ of 36?
47. What fraction of 36 is 15?
48. $\frac{2}{3}$ of $\frac{1}{6}$ of $\frac{3}{4}$ of a number is 12. What is the number?
49. Half the square of a number is 18. What is the number?
50. 81 is the square of twice what number?
51. Given a positive number x. Two times a positive number y is at least four times x. How small can y be?
52. Twice the square root of half a number is $2x$. What is the number?

Problems 53-55: A gathering has twice as many women as men. W is the number of women and M is the number of men:

53. Which is correct: $2M = W$ or $M = 2W$?
54. If there are 12 women, how many men are there?
55. If the total number of men and women present is 54, how many of each are there?

56. \$12,000 is divided into equal shares. Babs gets four shares, and Ben gets the one remaining share. What is the value of one share?

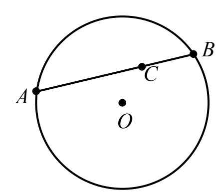
E. <u>Problems leading to two linear equations</u>:

57. Two science fiction coins have values x and y. Three x's and five y's have of 75¢, and one x and two y's have a value of 27¢. What is the value of each?
58. In mixing x *gm* of 3% and y *gm* of 8% solutions to get 10 *gm* of 5% solution, these equations are used:
$$.03x + .08y = .05(10), \text{ and}$$
$$x + y = 10$$
How many *gm* of 3% solution are needed?

F. <u>Geometry</u>:

59. Point x is on each of two given intersecting lines. How many such points x are there?
60. On the number line, points P and Q are two units apart. Q has coordinate x. What are the possible coordinates of P?

Problems 61-62:



61. If the length of chord AB is x and the length of CB is 16, what is AC?
62. If $AC = y$ and $CB = z$, how long is AB (in terms of y and z)?

Problems 63-64: The base of a rectangle is three times the height:

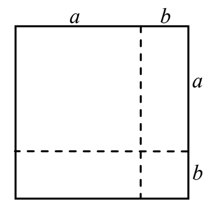
63. Find the height if the base is 20.
64. Find the perimeter and area.

65. In order to construct a square with an area which is 100 times the area of a given square, how long a side should be used?

Problems 66-67: The length of a rectangle is increased by 25% and its width is decreased by 40%.

66. Its new area is what percent of its old area?
67. By what percent has the old area increased or decreased?

68. The length of a rectangle is twice the width. If both dimensions are increased by 2 *cm*, the resulting rectangle has $84cm^2$ more area. What was the original width?
69. After a rectangular piece of knitted fabric shrinks in length 1 *cm* and stretches in width 2 *cm*, it is a square. If the original area was $40cm^2$, what is the square area?
70. This square is cut into two smaller squares and two non-square rectangles as shown. Before being cut, the large square had area $(a+b)^2$. The two smaller squares have areas a^2 and b^2. Find the total area of the two non-square rectangles. Show that the areas of the 4 parts add up to the area of the original square.



Answers:

1. 46
2. 9
3. 2
4. 1.08
5. 62.5%
6. 160%
7. 56.25
8. $581
9. 90%
10. 8000
11. lower
12. 96%
13. same (96%)
14. B
15. 125%
16. 80%
17. 65
18. 95
19. 68.2
20. .625 *amps*
21. .075 *kw*
22. .45 *kwh*
23. 13.5 *kwh*
24. $1.08
25. 450 *mph*
26. 2250 *mi*
27. $450x$ *mi*
28. $40/9$ *hr*
29. $y/450$ *hr*
30. 400 *mph*
31. $450-z$ *mph*
32. 3000 *mi*
33. $(450-z)t$ *mi*
34. $35/3$
35. $21/5$
36. 36
37. 576
38. $4:3$
39. $9/4$
40. x
41. $3:2$
42. y
43. $3:2$
44. 45
45. 48
46. 27
47. $5/12$
48. 144
49. 6
50. $9/2$
51. $2x$
52. $2x^2$
53. $2M=W$
54. 6
55. 18 men, 36 women
56. $1500
57. $x: 15¢$, $y: 6¢$
58. 6 *gm*
59. 1
60. $x-2$, $x+2$
61. $x-16$
62. $y+z$
63. $20/3$
64. $P=160/3$, $A=400/3$
65. 10 times the original side
66. 75%
67. 25% decrease
68. $5cm$
69. 49
70. $2ab$
    $a^2+2ab+b^2=(a+b)^2$